GENERAL SYNTHESIS OF OPTIMUM MULTI-ELEMENT COUPLED-TRANSMISSION-LINE DIRECTIONAL COUPLERS

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Coupled-transmission-line directional couplers of the simple quarter-wavelength type have theoretically perfect isolation and input match. The coupling varies sinusoidally with frequency, giving a useful bandwidth of approximately 2 : 1, but much greater bandwidths may be obtained by cascading sections. Thus for example a coupling of 3 db ± 0.4 db over a 5 : 1 band may be obtained using a symmetrical three-quarter wavelength coupler. However this is not optimum in the sense of having maximum bandwidth for a given coupling tolerance, owing to the restriction placed on the coupling function by making the two outer elements equal. When the three elements are allowed to be all unequal, then it is possible to obtain an optimum Tchebycheff coupling characteristic with two ripples. It may be shown that for an asymmetric n-element coupler, the general equation for the insertion loss to the straight through arm of the coupler is

$$L = 1 + \beta^2 - h^2 T_n^{\,2}\left(\frac{\cos\theta}{\cos\theta_o}\right) \quad \ldots \ldots \ldots \ldots \ldots \quad (1)$$

where T_n denotes the Tchebycheff function of the first kind of degree n, θ is the electrical length of each element, and β and h are constants. Defining $J = \cosh^{-1}\frac{\beta}{h}$, $H = \cosh^{-1}\sqrt{1 + \beta^2}/h$, then the pass band extends from θ_o to $\pi - \theta_o$

where $$1/\cos\theta_o = \cosh J/n \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2)$$

and the following expressions for the mean coupling C db and the coupling tolerance or ripple ± R db are obtained:-

$$C = 10 \log_{10} \frac{\sinh 2H}{\sinh 2J} \quad \ldots \ldots (3) \qquad R = 10 \log_{10} \frac{\tanh H}{\tanh J} \quad \ldots \ldots (4)$$

The bandwidth ratio $(\pi = \theta_o)/\theta_o$ as a function of the coupling tolerance of 3 db and 10 db couplers for a given number of elements is shown in Fig.1. A coupling of 3 db ± 0.4 db gives a bandwidth of 7 : 1 for a three-element asymmetric coupler, compared with 5 : 1 for the symmetric coupler. In addition to the optimization of bandwidth, the great advantage of the asymmetric coupler is that it is possible to synthesize the coupler to give a specified response. So far it has proved impossible to do this for the symmetric coupler, except for very loose coupling, when approximations can be applied.

A key to the synthesis of a directional coupler to give a specified coupling function, e.g. Equation (1), is given by the equivalence between the analysis of the coupler and that of a stepped-impedance filter. The coupler consists of n equal length sections with even-mode impedances (normalised to the input and output lines) of Z_r ($r = 1, 2 \ldots\ldots n$), and normalized odd-mode impedances given by the reciprocal of these values. It is then easily proved that the coupling and transmission coefficients of the coupler are identical to the reflection and transmission coefficients respectively of a stepped impedance filter consisting of a cascaded set of lines, each of electrical length θ, with impedances Z_r terminated by lines of unit impedance. The mathematical formulation of the four-port coupler problem is reduced to that of a two-port filter problem. This may be analysed by multiplication of the transfer matrices. The insertion loss derived from the overall matrix will be a polynomial in $\cos^2\theta$ of degree n, the coefficients being functions of Z_r. The synthesis may be performed by equating coefficients of this with the corresponding coefficients in Equation (1). This process leads to n simultaneous equations in the n variables Z_r which may be solved for values of n up to, say, $n = 3$. Beyond this the process rapidly becomes untractable.

A solution of the problem is obtained by modern synthesis techniques. The synthesis is performed using a theorem of Richards extended by Riblet* to the

* H.J. Riblet "General Synthesis of Quarter-wave Impedance Transformers", I.R.E. Trans., Vol.MTT-5, pp.36 - 43, January.1957.

synthesis of quarter-wave impedance transformers. It is first necessary to extend this theorem further to the synthesis of an insertion loss of the form of Equation (1). In fact, it is possible to generalize the theorem completely, and to state that any lumped-element two-port ladder network having all its poles at infinity has a stepped-impedance filter equivalent. Equation (1) has a lumped element prototype (obtained by substituting ω for $\frac{\cos\theta}{\cos\theta_o}$) which is physically realizable, and hence the corresponding stepped-impedance filter always exists.

Richards showed that the frequency transformation $t = \tanh \theta$, where θ is electrical line length, maps the usual complex frequency impedance plane into the plane $Z(t)$, where t is a new complex variable equivalent to the complex frequency variable in ordinary lumped element synthesis theory. Hence the reflection coefficient as a function of t must have no poles (or zeros, since it must be a minimum phase network) in the right half plane. Standard synthesis techniques are now applied to Equation (1) to give an expression for the reflection coefficient as a function of t. From this, the impedance function $Z(t)$ is derived, and hence the overall transfer matrix of the stepped impedance filter. The matrix is broken down into its n constituent matrices, leading to explicit formulas for the impedances Z_r as a function of J and H, which are related to the coupling, ripple, and bandwidth. The formulas for values of n up to $n = 5$ are given in the main paper, and it is possible to extend these to higher values of n.

So far the theory has been applied to design several two and three element couplers, and the agreement between theory and experiment has been excellent in all cases. Thus a two-element coupler has been made, using printed-stripline construction, with a coupling of 3.2 db $\pm$ 0.8 db over the band 155 - 1035 Mc/s, a 6.7 : 1 bandwidth ratio. The performance of a three-element coupler of 20 db $\pm$ 0.5 db for a 6 : 1 band is shown in Fig.2, which indicates how closely the double-ripple Tchebycheff characteristic is achieved.

The phase properties of the a symmetric couplers differ from the symmetric type previously considered. In the latter, the phase difference between the coupled and transmitted waves is 90° at all frequencies, but for the asymmetric coupler this phase difference varies almost linearly over the band. However by the addition of a length of line to one of the output arms, the phase variation can be approximatbly cancelled, and a usable 90° hybrid performance obtained from a 3 db coupler. In the case of a compensated 3 db $\pm$ 0.5 db coupler for a 5 : 1 band the phase difference over the band is $90^\circ \pm 22^\circ$, and variations of this order of magnitude are quite acceptable in many applications. In cases where the phase is required to be much more nearly equal to 90°, then it is desirable to use the symmetric coupler. This emphasises the importance of obtaining a solution to the problem of the synthesis of this type of coupler.

In conclusion, with the proof that a lumped-element ladder network forms a prototype for a stepped-impedance filter, it has been shown how to synthesize a class of asymmetric multi-element directional couplers to give optimum Tchebycheff coupling characteristics. The synthesis leads to explicit formulas for the design parameters, i.e. the normalised even-mode impedance of each coupler element, as a function of the bandwidth, coupling, and ripple. Two and three element couplers have been made and tested, giving excellent agreement with the theory, and it is now possible to design directional couplers for operation over bandwidths of one decade or more. The phase division is highly frequency dependent, but by careful choice of reference planes in the output ports, this can be made approximately 90° over the bandwidth of the coupler. The synthesis of symmetrical couplers remains unsolved.

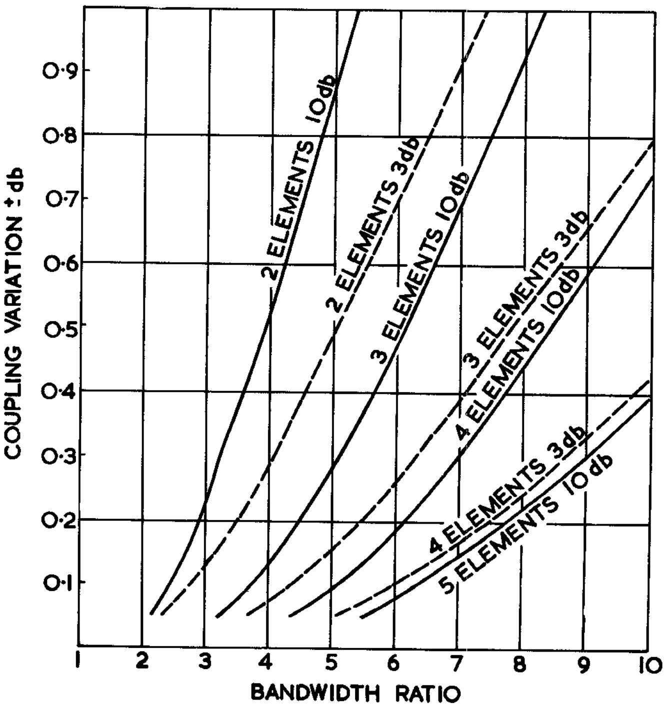

Fig.1. Coupling tolerance of 3 db and 10 db asymmetric stepped directional couplers as a function of bandwidth.

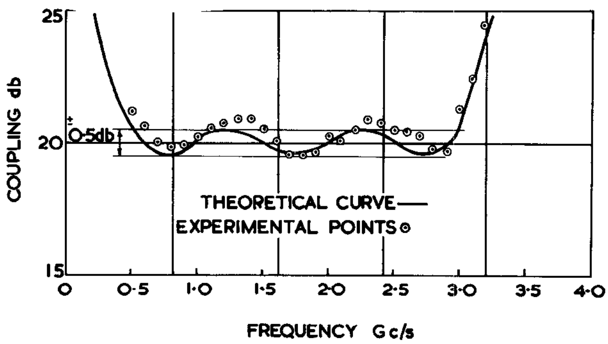

Fig.2 Three-element 20 db coupler.

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